

MAT 563 HOMEWORK 10: DUE FRIDAY, APR. 11

(1) In class we saw that sums, products, and quotients of algebraic elements are algebraic. Find the minimum polynomial (over \mathbf{Q}) of $\alpha = \sqrt{3} + \sqrt{5}$.

(2) Let K be a field, and consider an extension of the form $K(\alpha, \beta)$, where α and β are algebraic over K . Prove that

$$[K(\alpha, \beta) : K] \leq [K(\alpha) : K][K(\beta) : K].$$

Show also that equality holds if the integers $[K(\alpha) : K]$ and $[K(\beta) : K]$ are relatively prime.

(3) Suppose $[K(\alpha) : K]$ is odd. Prove that there exists a polynomial $f(x) \in K[x]$ such that $f(\alpha^2) = \alpha$. (Use the previous problem.)

(4) Suppose $\alpha \in \mathbf{C}$ is algebraic of degree n over \mathbf{Q} . Show that the real numbers

$$\operatorname{Re}(\alpha), \operatorname{Im}(\alpha), |\alpha|$$

are also algebraic. Give upper bounds for their degrees over \mathbf{Q} .

(5) Find the splitting field in \mathbf{C} of $f(x) = x^8 - 2$.

(Do not turn in the following- just for practice)

(6) Find all 8th roots of unity in \mathbf{C} and identify the primitive ones.

(7) Let $f(x) \in \mathbf{R}[x]$ be irreducible over the field of real number \mathbf{R} . Prove that $\deg f = 1$ or 2 .